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78

Theory of Diffusion Processes in Moving Fluids (Hydrodynamics of Large Prandtl Number)

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As is known, diffusion processes in moving fluids (convective diffusion) are substantially different in character from the process of convective heat conduction. This difference is connected to the difference in the values of the coefficient of diffusion and temperature conduction in fluids.

Namely, the diffusion coefficients of dissolved molecules and ions (in water) are usually $D \sim 10^{-5}$ sq. cm/sec.; the temperature conduction coefficient of water for room temperature equals $k \sim 1.4 \cdot 10^{-3}$ cm^2/sec., i.e., approximately 100 times larger than the diffusion coefficients of ordinary materials.

The transport of arbitrary substances (heat or material) in a moving fluid is usually characterized by the value of the nondimensional transport coefficient (Nusselt number). For the diffusion case

$$Nu_D = \frac{j_{diff} l}{D \Delta c}$$

where j_{diff} is the diffusion of the flow of matter through 1 cm^2 in 1 sec., l is the characteristic dimension and Δc is the difference in concentration near the body surface and in the depth of fluid.

In the case of heat conduction Nu_k has a similar form with D replaced by k and Δc by ΔT. The transport coefficient Nu is a function of the nondimensional Reynolds

number, Re , which characterizes the domain of fluid motion, and the Prandtl number, Pr , which characterizes the transport of matter in a moving fluid. The latter represents the ratio of the kinematic viscosity, ν, of the fluid to the diffusion coefficient D $(Pr_D = \nu/D)$ or the heat conduction $(Pr_k = \nu/k)$.

Hence

$$Nu = f(Re, Pr)$$

The difference in the numerical values of the diffusion coefficient and the temperature conduction leads to the Prandtl criterion having completely different values in the case of diffusion $(Pr_D \gtrsim 10^3)$ and temperature conduction $(Pr_k \sim 1-10)$. This is the difference in the transport processes in fluids and gases. In the latter, both Prandtl criteria-for diffusion and for temperature conduction- have values of the order of unity. From the above, there results that the process of convective diffusion in fluids may be characterized as the process of high Prandtl number, $Pr \gg 1$.

The questions of convective diffusion have substantial value in many physico-chemical and engineering processes. Thus, the convective diffusion of ions in electrolytic solutions determines the value of the current density which may flow through the electrolyte to the electrode [1]. The speed of heterogeneous chemical processes occurring at the dividing surface of a fluid-solid body, fluid-gas, fluid-fluid (for example, adsorption process, chemical conversions on surfaces in the presence of a catalyzer, etc.) is determined in most cases by the rapidity

* Numbers in brackets refer to references.

of material transport (or of one of the materials figuring in the reaction process) to the place of the reaction, i.e., the process of convective diffusion. Finally, the speed of solubility of the majority of solid bodies and many gases in fluids is determined by the speed of the removal of the dissolved material from the **separation boundary** on which the solubility process is taking place.

In a group of works [1-4] we evolved a quantitative theory of the process of convective diffusion in fluids related to the above-mentioned physicao-chemical questions.

In the present note, we want to emphasize greatly and to turn attention to the applicability of the analogy between diffusion and heat conduction, so widely used in gases, to the case of fluids.

It is shown in the theory of heat transfer in a moving fluid that the heat transfer process for $Re \gg 1$ is limited by the distribution of heat through a thin boundary layer which is generated in fluids near the surface of the flow around solid bodies. In this layer, called usually the thermal layer, occurs the fundamental temperature jump ΔT.

In all calculations of the theory of heat transfer, there is used the fact that for this process the Prandtl number may be considered ~ 1. This permits identifying the thermal layer with the hydrodynamic boundary layer.

Otherwise, in the limit case $Pr \gg 1$. In this case, there is also generated near the separation surface a diffusion boundary layer in which the basic variation in concentration Δc

occurs. However its thickness is found to be considerably less than the thickness of the hydrodynamic boundary layer.

As is known, the value of the thickness of the boundary layer is determined to an order of magnitude by the condition of equivalence of the molecular and convective flow of a substance. In the case of the hydrodynamic boundary layer, the transport of a substance is the momentum. Consequently, the thickness, δ, of the hydrodynamic boundary layer is determined by the condition

$$\eta \frac{v_x}{\delta} \sim \rho v_x v_y \sim \rho v_x^2 \frac{\delta}{l} \tag{1}$$

which reduces to the well-known expression

$$\delta \sim \sqrt{\frac{\nu l}{v_x}} \tag{2}$$

where v_x is the characteristic velocity of the flow and l is the characteristic dimension.

The condition of equivalent molecular and convective flow of matter may be written analogously for the diffusion boundary layer

$$D \frac{\Delta c}{\delta'} \sim v_y \Delta c \tag{3}$$

However, inasmuch as $D \ll \eta/\rho = \nu$, equation (3) is valid at distances δ' from the wall which are much less than δ, i.e., within the hydrodynamic boundary layer. Here $v_y \sim \nu y^2/\delta^3$. Consequently

$$\delta' \sim \frac{D}{v_y} \sim \frac{D\delta^3}{\nu \delta'^2}$$

$$\delta' \sim \left(\frac{D}{\nu}\right)^{1/3} \delta \sim \frac{1}{10} \delta$$

Therefore, the thickness, δ', of the diffusion boundary layer comprises only about 0.1 the thickness of the hydrodynamic boundary layer δ. The latter fact distinguishes, in a radical manner, the diffusion boundary layer from the thermal which has a thickness almost exactly equal to δ.

The preceding reasoning is related to the case of the laminar regime of fluid flow. If the flow in the hydrodynamic boundary layer or in a pipe, in the case of the internal problem, becomes turbulent, then additional considerations are necessary.

In the case of turbulent flow there is the well-known Reynolds analogy establishing the similarity between the phe - nomena of momentum transport and heat ([5] page 329 and [6,7]). The technique often used of dividing the turbulent flow into a turbulent region and a laminar sublayer in which the turbulence vanishes completely, raises the objection that, in the main, there is no basis whatever for the sudden vanishing of turbulent pulsation and from the point of view of experimental data concerning the existence of turbulent pulsations of pressure on the surface of a solid body by the study of velocity profiles and data on heat transfer. This fact prompted many authors to express doubts apropos the existence of turbulence up to the wall itself [8,9]. However, no matter what the law of decreasing turbulence close to the wall, it is always possible to introduce a sublayer of fluid in which the momentum transport by a molecular (viscous) mechanism exceeds its transport by turbulent pulsations. Namely, this sublayer

of thickness δ_o is called the viscous sublayer. It is clear from the character of its determination that there exist in the viscous sublayer, although weakened, turbulent pulsations. The law of decrease of turbulence with distance, y , from a wall is, in the viscous layer: $\lambda \sim \text{const.}y^2/\delta_o$, where λ is the scale of turbulent pulsations [1,10].

These turbulent pulsations, transporting a small share of the general momentum flow, are found to be still sufficiently powerful to transport the principal share of the matter inasmuch as $D \ll \nu$. Therefore, the whole turbulent flow is divided into three regions:

Region	Basic Mechanism of Momentum Transport	Basic Mechanism of Material Transport
$y < \delta'$	Viscosity	Molecular diffusion
$\delta' < y < \delta_o$	Viscosity	Turbulent pulsation
$\delta_o < y$	Turbulent pulsation	Turbulent pulsation

The case of convective material transport at the separation boundary of fluid-fluid also requires special consideration. The tangential component of the stress tensor remains continuous on this boundary, but the velocity does not go to zero. Thanks to this, the similarity between momentum transport and material transport is completely upset.

We calculated the Nusselt number Nu (for $Pr \gg 1$) in a series of cases (see Table 1).

The calculation of Nu for a rotating disc was verified experimentally by B. N. Kabanov & U. G. Siver [11]. Accord with the tests is found to be excellent. The formulas for

Nu in the case of turbulent flow around a flat plate is also found to be in accord with tests.

Table 1

Reynolds Number	Flow regime	Geometric condition	Transport Coefficient
$\dfrac{uR}{\nu} \ll 1$	Viscous flow	Surface of spherical drop	$2.04 \, Pr^{1/3} Re^{1/2}$
$\dfrac{uL}{\nu} \gg 1$	Laminar boundary layer	Stream flow over plate of length L	$0.67 \, Pr^{1/3} Re^{1/2}$
$\dfrac{\omega R^2}{\nu} \gg 1$	Same	Rotating disc	$0.617 Pr^{1/3} Re^{1/2}$
$Gr \gg 1$	Same	Natural convection	$0.73 \, Pr^{1/4} Gr'$
---	---	Smooth surface of draining film	$0.63 \, Pr^{1/2} Re^{1/2}$
$\dfrac{uL}{\nu} \gg 1$	Turbulent flow	Surface of plate	$Pr^{1/4} \dfrac{Re}{\ln Re}$ (logarithmic accuracy)
$\dfrac{uL}{\nu} \gg 1$	Same	Surface of draining fluid film	$Pr^{1/2} Re^{3/4}$

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